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Impact of using virtual and concrete manipulatives on students' learning of fractions

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ABSTRACT

Fraction is considered one of the main topics in mathematics in the early years. Many students struggle to progress in fractions in the classrooms and on exams. Teaching strategies are among the most important factors affecting student learning of fractions. The objective of this study was to examine the effectiveness of virtual and concrete manipulatives for teaching and learning fraction addition and subtraction to fifth-grade students in selected private schools in Abu Dhabi in the United Arab Emirates. The study employed a quasi-experimental approach in which 81 students (40 male and 41 female) in four groups of grade-five participated in the pretest, intervention, and posttest. The intervention was provided by teaching fraction addition and subtraction by using concrete and virtual manipulatives. Inferential statistics such as Wilcoxon Signed Ranked test, Mann-Whitney U test and Kruskal Wallis test were employed. The results indicated that student achievement in learning fractions using concrete manipulatives was better than virtual manipulatives. Also, girls' progress in learning fractions was more significant than boys. The findings of this study could help mathematics education by enhancing teaching strategies that have significant implications for improving students' fraction computation skills.

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Introduction

In the early years of education, students are taught fractions, which is regarded as one of the key topics in mathematics. The National Council of Teachers of Mathematics (NCTM) stated that 'Beyond understanding whole numbers, young children can be encouraged to understand and represent commonly used fractions in context, such as $^1/_2$ of a cookie or 1/8 of a pizza, and to see fractions as a part of a unit whole or of a collection' (NCTM, 2000, p. 33). The NCTM further emphasizes that mathematics teachers should develop their students' understanding of fractions as dividing numbers, proportionality, and decimals.

The school mathematics curricula and standards have placed fractions as critical mathematical thinking and reasoning skills. Many scholars agree that fractions are part of everyday life that students deal with and use in problem-solving (Hansen et al., 2017; Kocaoglu, 2010). However, many students need help learning fractions (Aksoy & Yazlik, 2017; Bentley & Bossé, 2018). Also, it is considered one of the most challenging topics for elementary teachers to teach (Kor et al., 2018). Students need help grasping the concept due to confusing fractions with whole numbers (McIntosh et al., 1992) and misconceptions (Abdullah et al., 2018; Alghazo & Alghazo, 2017; Aliustaoğlu et al., 2018; Pesen, 2007; Şiap & Duru, 2004;

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Yilmaz et al., 2018). In particular, students' misconceptions might have developed over time due to inappropriate teaching by teachers (Baki, 2008; Hansen et al., 2017). As such, different teaching approaches can be implemented to overcome these challenges and improve the teaching and learning of fractions.

Teaching and learning with virtual manipulatives

Bouck and Flanagan (2010) defined virtual manipulation as 'computer-based simulations of physical manipulatives that are accessed via the internet or computer software' Dorward (2002, p. 186) described the virtual manipulative as renditions of actual manipulatives and tools to the computer. In the post-modern era, technology has entered all fields, including education. As a result, there are many calls to integrate technology with teaching, and thousands of educational applications have been created for teaching and learning mathematics. For example, some studies assert that integrating GeoGebra with mathematics encourages and trains students with the needed skills to understand and do mathematics (Moeller & Reitzes, 2011; Velichova, 2011). Kramarenko et al. (2019) stated that since technology has an important place in innovative development, it can have considerable potential for implementation in mathematics learning. However, applications such as GeoGebra, Matlab, and Desmose are only a few applications used for teaching and learning mathematics with modeling and simulations. It is crucial to review and test online digital tools and applications for effectiveness and usefulness to enhance the teaching and learning process.

The realities of the modern educational environment require a digital teacher who includes software games in educational and cognitive mathematical activities. To attract the attention of students, the developers of technological applications for learning mathematics use many approaches, such as computer games, and electronic manipulatives with colorful and lovely graphics. In addition, to engage all the students with different levels of thinking, those applications should include Bloom's taxonomy - thinking skills, from memorizing to synthesizing, evaluating, predicting, and innovating.

Virtual manipulatives help students to increase imagination, construction, and problem-solving skills (Donevska-Todorova et al., 2020; Ernest, 2010). Moreover, developing online teaching materials can help students to study anywhere and anytime. Virtual manipulatives could create new complex problems that rely on critical and computational thinking (Voskoglou & Salem, 2020). On the other hand, using virtual manipulatives could distract the students. Computers might not solve problems for students. Instead, technology could help students develop operational skills, critical thinking, and creative reasoning. Thus, students could be trained not to depend entirely on technology to solve calculations; instead, they have to practice and solve questions mentally to develop their mathematical thinking skills (Voskoglou & Salem, 2020).

A large body of research has suggested using manipulatives in teaching fractions to enable students to grasp concepts quickly (Ukdem & Çetin, 2022). Teaching fractions using virtual manipulatives allows the students to learn fractions through trial and error (Parungao, 2021). For example, students can use online tiles to model fractions and perform operations on fractions or compare and arrange fractions in ascending or descending ways. Students can model real-world problem-solving using virtual manipulatives that are cheap and costless, and there is no time needed for preparations. In the current study, virtual manipulation is considered a critical approach to teaching fractions to students in the UAE.

Past studies focused on different problems related to use of virtual manipulatives. For example, Park et al. (2022) examined the effectiveness of virtual manipulatives while teaching students with learning disabilities. The results from review of several studies showed an improvement in students' mathematics learning with virtual manipulatives. Likewise, Bouck et al. (2017), Bouck & Park (2018), and Bouck, Park et al. (2020) used virtual representational tools and applications as assistive technology for teaching equivalent fractions to students with disabilities. The application of virtual manipulatives in normal classrooms were examined in Keldgord and Ching (2022) with teachers' views. These studies were conducted on special need students' classrooms under experimental settings or on special circumstances, such as COVID-19 pandemic. However, there are limited studies on the use of virtual manipulatives in normal classrooms with an objective compare their effects on student learning compared to other tools, such as concrete manipulatives, especially in Arabian context.

Teaching and learning with concrete manipulatives

Another important strategy that helps mathematics teaching is using concrete manipulatives such as cubes and solid shapes to find the area and calculate the missing parts using the Pythagorean theorem. The importance of using concrete materials in teaching and learning comes from the third stage of Piaget's four stages of cognitive development. The concrete operational (Ages 7 to 11) is using models and learning by doing (Piaget, 1977). In general, using concrete manipulatives and notes in instructional strategies is often suggested as an effective method to improve students' mathematics achievement (Carbonneau et al.). Having students use concrete manipulatives and notes and undertaking mathematical projects with manipulatives will allow them to physically interact with objects from their real life to learn new concepts about mathematics, which is more helpful and productive (NCTM, 2000).

Marshall and Swan (2008) demonstrated some advantages and disadvantages of using concrete manipulatives in teaching and learning mathematics in their experimental study. Some examples of the advantages include but are not limited to; assisting in concrete visualization, they provided hands-on learning, building better understanding, engaging students, providing them with enjoyment, providing motivation, give a chance to grasp concepts, appealing to all styles of learning, encourages oral language, provide opportunities for collaborative learning to occur, allow for peer tutoring, and teacher can more easily note what the child is thinking through the use of manipulatives. On the other hand, there are some disadvantages: teachers often do not have enough objects and materials for all the students, a problem having the materials ready to use with easy access, particularly when sharing with other teachers, the cost of manipulatives made their use problematic, children fiddling and not listening to instructions that they were often a distraction, the amount of noise generated with their use, behavior management, equipment misuse, storage and organization of the materials, not having enough time to teach using manipulatives, and time to organize, set up and pack them away, can be used for play without proper instruction, hygiene issues, and safety that students could put pieces in the mouth (Marshall & Swan, 2008). Regardless, the advantages seemed to outweigh the disadvantages of manipulatives necessitating concrete manipulatives in mathematics teaching (Lee & Chen, 2015).

Comparison of concrete and virtual manipulatives

There are separate studies on the effectiveness of using virtual manipulatives and concrete manipulatives in teaching and learning mathematics. For example, on one hand, Bouck and Flanagan (2010), Donevska-Todorova et al. (2020), Moyer-Packenham and Westenskow (2013), Parungao (2021), Park et al. (2022), and Shin et al. (2016) examined applications of virtual manipulatives in teaching mathematics. On the other hand, Carbonneau et al., Flores et al. (2018), Lee & Chen (2015), Marshall & Swan (2008) and Pratt & Eddy (2017) examined the effectiveness of concrete manipulatives in modeling concepts and teaching mathematics. However, a few studies compared the effectiveness of virtual and concrete manipulatives in teaching and learning fractions or other math concepts. For example, Bone et al. (2021), Mendiburo and Hasselbring (2014), Moyer-Packenham et al. (2013), Rosen and Hoffman (2009), and Sarama and Clements (2016) compared the effectiveness of concrete and virtual manipulatives in teaching and learning mathematics. There are no conclusive results in terms of which one is better than the other. Rather, it depends on which content and context of learning these manipulatives support. However, such studies are limited to Western educational contexts. The current study is new in the Arab context in general and in the United Arab Emirates in particular.

Context of the study

Despite being prevalent in daily life, many students still find fractions challenging to grasp (Fazio & Siegler, 2011). Traditional teaching methods alone may not be effective and could lead to students developing a dislike for the subject (Wijaya et al., 2020). Additionally, according to cognitive theory, fractions are typically taught to students aged between 10 and 12 years old, who are learning by doing rather than memorizing or abstract thinking (Pulaski, 1971). Therefore, to effectively engage all students, teachers should vary their teaching methods by incorporating technology and modeling techniques and design activities based on their levels, cultures, and interests.

Based on the PISA 2018 rankings, the United Arab Emirates placed 50th out of 78 countries in mathematics (OECD, 2018). Furthermore, in the TIMSS 2015, the United Arab Emirates was ranked 39th out of 49 countries for grade four and 23rd out of 39 for grade eight mathematics (Mullis et al., 2016). Students often struggle with learning fractions, which may be attributed to the teaching methods employed (Pantziara & Philippou, 2012). There needs to be more research on appropriate fraction teaching methods in the UAE, lending support for this experimental study to explore virtual manipulatives and concrete manipulatives as teaching strategies for fractions.

The use of virtual or concrete manipulatives may support constructivist teaching and learning that may help in critical and constructive learning of mathematics in general and fractions in particular from real experience (Doolittle, 1999; Kalina & Powell, 2009). While doing this, students and teachers work together as collaborators in the learning process for creativity and critical thinking against banking pedagogy (Freire, 1972).

This study aims to examine the impact of teaching fractions using virtual manipulatives and concrete manipulatives on fifth-grade students' learning of fraction addition and subtraction. To achieve the purpose of this study, we answered the following research questions:

1. What is students' performance when learning fractions, addition, and subtraction using virtual and concrete manipulatives?
2. Do gender disparities exist in the acquisition of fraction concepts when utilizing virtual manipulatives versus concrete manipulatives?
3. Does the teaching strategy employed with virtual and concrete manipulatives affect students' achievement in learning fractions?

Five null hypotheses have been formulated based on the independent variable of using either virtual manipulatives or concrete manipulatives and the dependent variables of students' progress in learning fraction addition and subtraction.

H_0 #1 (Q1) There is no statistically significant difference in student achievement from the median value 1 (partially correct) in fraction addition and subtraction while teaching by using virtual manipulatives at a significance level of $\alpha = 0.05$.

H_0 #2 (Q1) There is no statistically significant difference in student achievement from the median value 1 (partially correct) in fraction addition and subtraction while teaching by using concrete manipulatives at a significance level of $\alpha = 0.05$.

H_0 #3 (Q2) There is no statistically significant difference at $\alpha \leq 0.05$ between male and female students in learning fraction addition and subtraction when using virtual manipulatives at a significance level of $\alpha = 0.05$.

H_0 #4 (Q2) There is no statistically significant difference between male and female students in learning fraction addition and subtraction when using concrete manipulatives at a significance level of $\alpha = 0.05$.

H0 #5 (Q3) There is no statistically significant difference in students' achievement in fraction addition and subtraction by teaching with virtual and concrete manipulatives at a significance level of $\alpha = 0.05$.

Methodology

This study applied a quasi-experimental one-group pretest-posttest design (Ellington et al., 2020; Flannelly et al., 2018), without random assignment of the participants in the experimental and control groups (Martin & Bridgmon, 2012). The sample study involved 81 grade-five students (40 male and 41 female) divided into four groups. Participants sat for a pretest, received an intervention, and then took a posttest. The intervention involved teaching fraction addition and subtraction using both concrete and virtual manipulatives. Inferential statistical methods, including the Wilcoxon Signed Rank test, Mann-Whitney U test, and Kruskal-Wallis test, were applied to analyze the data.

Sample of the study

We conducted our study in four schools, two located in Al Ain city and two in Abu Dhabi city, out of the 516 private schools in Abu Dhabi during the 2022–2023 academic year, as reported by the Abu

Dhabi Department of Education and Knowledge (ADEK, 2023). Three schools followed the United Arab Emirates national curriculum, while one followed the American curriculum. The sample schools were selected through non-random sampling based on the researcher's good relationship with the school principals, which made the process easier. Furthermore, the schools had different mathematics curricula. They were situated in different locations within the Emirate of Abu Dhabi, making them a suitable sample for studying the impact of the two mathematics teaching strategies on the students' fractions learning. As the schools were in different cities, communication between students and teachers from different schools was limited, thus ensuring that the results were not influenced by such communication. The study focused on grade five students, with varying numbers of participants in each of the four schools (as illustrated in Table 1).

Setting and implementing the study

In the second term of the academic year, both the UAE MOE and American curriculum introduce subtracting like fractions and adding unlike fractions to grade five mathematics students. This means that students from both curricula learn subtracting like fractions and adding unlike fractions simultaneously. Each lesson requires two periods to complete, each period was of about 50 minutes. The total number of periods was four, with two periods allocated for each lesson. The researchers contacted the mathematics teachers in the four schools of the study and trained them on the teaching strategies. The lesson plan and the resources used were unified across the four schools. Virtual manipulatives were used to teach fraction subtraction of like fractions, while concrete manipulatives were used to add, unlike fractions. Table 2 outlines the lesson order and relevant standards. Teachers were instructed to use the same online mathematics applications for virtual manipulatives, and real objects such as fractions, bars, and circles were used for concrete manipulatives. The strategies were applied for two periods each.

Teaching fractions with virtual manipulatives

For the first teaching strategy, which involved teaching subtracting like fractions using virtual manipulatives, the researcher opted for specific online mathematics applications such as Unifix Cubes from Didax.com and Fraction Strips from Toy Theater, which allowed students to create models for subtracting fractions with like denominators and perform other fraction operations. Although there are numerous online applications available, the researcher selected these specific ones to ensure consistency in teaching across all four schools. These virtual tools enabled students to explore mathematical concepts and solve real-life problems, and they were easily accessible, time-efficient, and cost-effective. In this strategy, teachers operated the applications via a computer while allowing students to use tablets, laptops, or other devices to solve fraction problems (refer to Figures 1a and 1b). As a result, students learned about fractions by constructing models virtually, which made the learning process enjoyable and self-directed. This method serves as an alternative to using concrete manipulatives.

Teaching fractions with concrete manipulatives

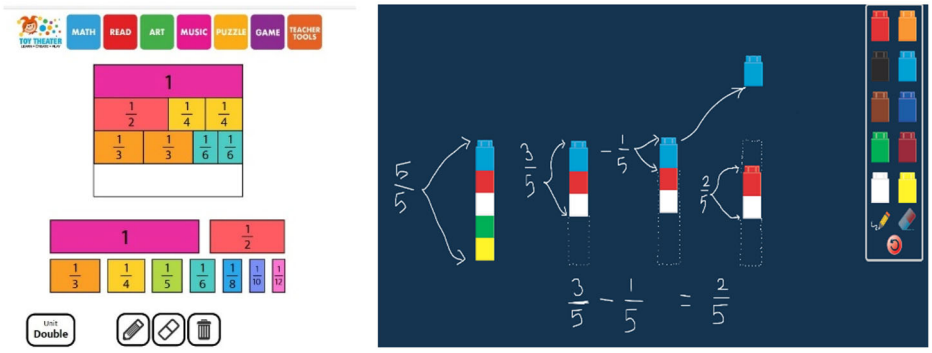
For the second strategy involving adding unlike fractions, the researcher provided mathematics teachers with various concrete manipulatives, such as magnetic fraction tiles, fraction circles, fraction cubes, wooden fractions, and magnetic pizza fractions, that were chosen for their appropriateness in teaching fractions. In about ten minutes, the teacher explained how to use concrete manipulatives to add unlike fractions, providing some examples. Afterward, the students worked in groups to solve similar questions of varying difficulty levels. By using these concrete objects, students were able to better comprehend

Table 1. Distribution of students in 4 schools.

		No. of Students participated		
School	Curriculum	Male	Female	Total
IKS	UAE MOE	7	4	11
DUF	UAE MOE	12	10	22
DUY	UAE MOE	13	21	34
UPSY	American	8	6	14
Total		40	41	81

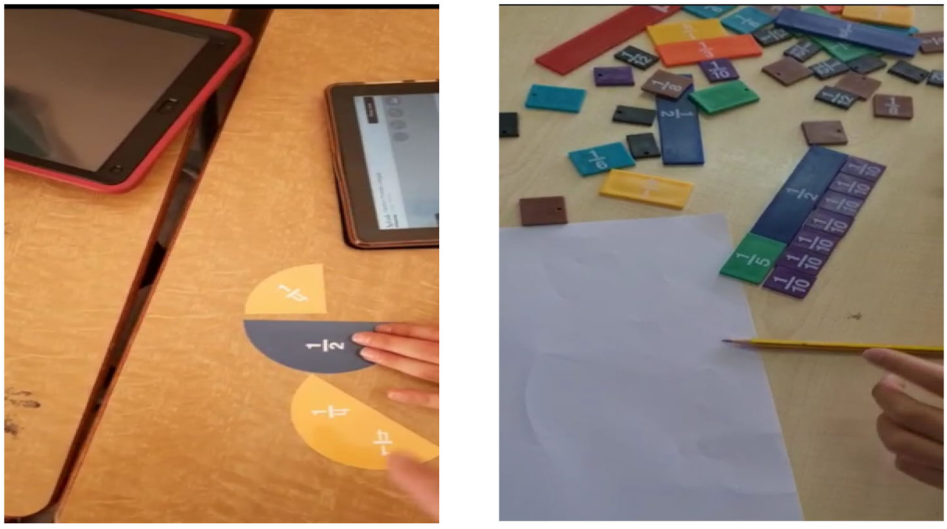
Table 2. Curriculum of fraction addition and subtraction for grade-5 (UAE MOE - American Curriculum).

5th-grade Mathematics Plan adding and subtracting fractions		
MOE UAE	Standard	No. of needed periods
Unit 8: Lesson 3: Subtracting like fractions Lesson 6: Adding unlike fractions	5.9.2.1	2
American Chapter 6: L6.2 Subtraction with like denominators L6.5 Addition with unlike denominators	CCSS 5.NF.1	2



(a) (b)

Figure 1. Using virtual manipulatives in teaching fractions (Didax, 2023; Toy Theater, 2023).



(a) (b)

Figure 2. Using concrete manipulatives in teaching fractions.

the abstract concepts of fractions and fraction operations. They modeled and solved questions related to adding unlike fractions using these manipulatives (refer to Figure 2). It was crucial for students to compare fractions and model equivalent fractions to be able to add, unlike fractions. For instance, when adding 1/2 and 1/4, students may initially assume that 1/4 is larger than 1/2 because 4 is greater than 2 (Figure 2a). However, by using fraction bars or circles, they would realize that 1/2 is in fact greater than 1/4. To add 1/2 and 1/4, they would represent 1/2 as one part consisting of 2 pieces of 1/4, resulting in a final answer of 3/4. Therefore, through the construction of fractions using concrete manipulatives, students learned about comparing fractions, equivalent fractions, and adding fractions, among other concepts. They worked on several other examples involving modeling of fraction addition and subtraction (e.g. Figure 2b).

Development of assessment tool

The researcher developed an assessment tool consisting of six questions and two problem-solving questions, with a time limit of 20 minutes, focused on subtracting like fractions and adding unlike fractions. The questions were carefully selected to cover the objectives of each lesson according to the MoE pacing guide. The assessment tool was administered as a pretest to all students in the four schools. Following the implementation of the lesson on subtracting like fractions using virtual manipulatives, the students took a posttest that included the same questions as the pretest. Similarly, after the second lesson on adding unlike fractions using concrete manipulatives, the students took a posttest that featured questions related to adding unlike fractions (refer to Table 3).

The aim of administering pre and posttests was to evaluate the students' learning progress to address the research questions.

Validating the tools

In order to establish the validity and reliability of the assessment instrument, the researchers first selected 6-short answer questions and 2-word problem questions for the pre and posttests from both the United Arab Emirates and American grade five mathematics textbooks, which were aligned with the UAE MOE mathematics standards and the common core state standards (CCSS). Next, the questions were reviewed by four experienced mathematics teachers to evaluate their suitability and difficulty level. Based on their feedback, the questions were modified accordingly. Table 4 shows some examples of the pre and posttests.

The researcher obtained ethical approval for the study from the Research Ethical Committee (REC) at a higher education institution in the UAE. After writing the eight test questions in both English and Arabic, the researcher sought the evaluation and validation of the questions from ten mathematics teachers and education experts from four countries: the United States of America, the United Arab Emirates, Nepal, and Palestine. The researchers obtained their permission to validate the tools and sent them an email with the questions. The evaluators were asked to assess four criteria: language, content, diagrams, and translation (for bilingual evaluators). Their feedback was highly informative and helped to refine the questions. For example, one of the reviewer's comments on the question 'Use the figure to find the sum of a half and one-third and complete the addition sentence' was 'add the clear direction' The comment was accepted, and the question was modified to make it clear. The instrument was modified according to their recommendations and was deemed suitable for use.

The reliability of the instrument was established by assessing the internal consistency of the items. This resulted in Cronbach's alpha value of 0.865, which was deemed to be appropriate for this study (Norcini, 1999).

Data analysis and interpretation

This quasi-experimental study utilized a non-random pretest-posttest model to evaluate the impact of virtual and concrete manipulatives strategies on Grade 5 students' understanding of fractions. The pretest was administered before the study as a baseline measurement, while the posttest was given after each teaching strategy to assess the student's progress (Polit & Beck, 2013). The results were recorded on a Microsoft Excel spreadsheet and subsequently analyzed using the Statistical Package for the Social Sciences (SPSS) version 28.0 for statistical analysis (Collins, 2020). The data were first subjected to a normality test for the distribution of average scores of students in pre and posttests. Then, based on the normality test results,

Table 3. Types and number of items in pre- and posttests.

	Pretest		Posttest	
Strategy	Short Answer Questions	Problem-Solving	Short Answer Questions	Problem-Solving
Virtual Manipulatives	3	1	3	1
Concrete Manipulatives	3	1	3	1
Total	8		8	

Table 4. Samples of the pre and posttest Questions.

Subtracting like fractions	Adding unlike fractions
Show the difference by shading the figure and completing the addition sentence.  $\frac{5}{6} - \frac{1}{6} = =$ ___	Find the Sum $\frac{1}{5} + \frac{5}{10} =$
Find the Difference between seven tenths and four tenths	Shaikha bought white chocolate and brown chocolate for her friends to celebrate teacher day at the school. $\frac{1}{4}$ of her friends chose the white chocolate while $\frac{3}{8}$ chose the brown chocolate. Write the fraction that represents the total number of students who took chocolate?

Table 5. Test of normality for the pre and posttest.

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
Item	Statistic	df	Sig	Statistic	df	Sig
PreRVMAv	.284	59	<.001	.762	59	<.001
PreRCMAv	.479	59	<.001	.445	59	<.001
PostVMAv	.200	59	<.001	.856	59	<.001
PostCMAv	.204	59	<.001	.834	59	<.001

[a]Lilliefors Significance Correction.

non-parametric tests such as one-sample Wilcoxon Signed Ranked Tests, groups, and methods comparisons with Mann-Whitney (Martin & Bridgmon, 2012) and Kruskal Wallis tests were employed.

Results

To determine whether the data in this study followed a normal distribution, the researcher examined the responses of 59 participants to the pre and posttest questions and the average achievements of students in the tests by using Kolmogorov–Smirnov test and Shapiro–Wilk tests (Martin & Bridgmon, 2012). The results are displayed in Table 5.

Table 5 shows the Kolmogorov-Smirnov and Shapiro-Wilk tests of normality for the pretest and posttest. We code the items as follows: PreRVMAv and PreRCMAv mean the average of the pretest scores related to virtual manipulatives and concrete manipulatives strategy, respectively, and PostVMAv, PostCMAv mean the average of the posttest scores after virtual manipulatives and concrete manipulatives strategy method were implemented respectively. The results indicated that the data did not follow a normal distribution. Therefore, non-parametric tests were employed. Specifically, the Wilcoxon test, a distribution-free statistical tool, was used to evaluate whether the questions were derived from a population that was symmetric and had a specified median. Non-parametric statistical tests are distribution-free, which means that normality and homogeneity of variance assumptions are not necessary. Furthermore, the Independent-Samples Mann-Whitney U Test was utilized to compare groups based on gender.

One-Sample test (pretest) for the virtual and concrete manipulatives strategies

In this study, the student's results for each question were compared to a median (the test value) of 1. This is because the marks assigned to the students' answers for the pretest questions were 0 (not correct), 1 (partially correct), and 2 (correct).

Pretest for virtual manipulatives

Table 5 shows the one-sample Wilcoxon signed rank test for four pretest questions related to virtual manipulatives. For the first question, 'Show the difference by shading the figure and completing the

addition sentence', the results ($p < 0.001$, $z = -5.431$) showed a statistically significant difference from the test value (median = 1, knowing that 0: incorrect answer, 1: partially correct, and 2: correct), and the z-value is negative. Thus, students could not answer the first question. The results of the second question, which asks students to 'Find the Difference between seven tenths and four tenths', indicated that the result is not statistically significantly different from the test value ($p = 0.061 > 0.05$, $z = -1.871$) and since the z-value is negative, it is concluded that students slightly could answer the second question (See Table 6).

In the third question, where students were required to 'Find the difference of 4/5 - 1/5 - 2/5', the results ($p < 0.001$, $z = -5.598$) illustrated a statistically significant difference from the test value, which means that students failed to answer the third question. The results of the last question that requires writing a fraction as follows 'Bilal ran the 5/7 of the distance in a race before stopping for a break. At the same time, Hessa ran 3/7 of the distance in the same race. Write the fraction that represents the difference between Bilal's and Hessa's distance in the race', the result was statistically significantly different from the test value ($p < 0.001$, $z = -7.020$). Since the z-value is negative, therefore, the students could not answer the fourth question. The results of the average (PreRVMAv) of the second four pretest questions related to learning fractions (which had already been done by virtual manipulatives during the experiment) were statistically significantly different from the test value ($p < 0.001$, $z = -5.860$) and the z-value is negative, which indicates that the students, in average, failed to answer the four pretest questions related to virtual manipulatives strategy (See Table 5).

Pretest for concrete manipulatives

In Table 6, the one-sample Wilcoxon signed rank test was performed for four pretest questions related to the third variable (Concrete manipulatives). For the first question, when students were asked to 'Find the Sum $1/5 + 5/10$', the results ($p < 0.001$, $z = -7.091$) showed a statistically significant difference from the test value of 1, and the z-value was negative. Thus, students could not answer the first question. The results of the second question, which requires students to 'Use the figure to find the sum of a half and one third and complete the addition sentence', indicated that there is a statistically significant difference from the test value ($p < 0.001$, $z = -6.119$) and since the z-value is negative, it is concluded that students could not answer the second question. In the third question, which asks students to 'Find the Sum in simplest form $11/12 + 2/3 + 1/4$', the results ($p < 0.001$, $z = -6.755$) illustrated a statistically significant difference from the test value, which means that students failed to answer the third question (Table 7).

The results of the last question that requires students to write a fraction in response to 'Shaikha bought white chocolate and brown chocolate for her friends to celebrate teacher day at the school. 1/4 of her friends chose the white chocolate while 3/8 chose the brown chocolate. Write the fraction that represents the total number of students who took chocolate', this was statistically significantly different from the test value ($p < 0.001$, $z = -6.490$). Since the z-value is negative, therefore, the students could not answer the fourth question. The results of the average (PreRCMAv) of the third four pretest questions related to learning fractions (which were done by concrete manipulatives during the experiment) were statistically significantly different from the test value ($p < 0.001$, $z = -7.034$) and the z-value is negative, which indicates that the students, in average, failed to answer the four pretest questions related to concrete manipulatives strategy (See Table 7).

Posttest for virtual manipulatives

A one-sample Wilcoxon signed rank test was performed for four posttest questions related to virtual manipulatives. The results are shown in Table 8. For the first question, which requires students to 'Show

Table 6. One-sample wilcoxon signed rank test summary for virtual manipulatives (Test Value = 1).

Item	Pre1	Pre2	Pre3	Pre4	PreRVMAv
Total N	59	59	59	59	59
Test Statistic	232.000	598.500	240.000	58.000	50.000
Standard Error	109.473	106.637	115.217	109.473	100.259
Standardized Test Statistic	−5.431	−1.871	−5.598	−7.020	−5.860
Asymptotic Sig.(2-sided test)	<.001	.061	<.001	<.001	<.001

Table 7. One-sample wilcoxon signed rank test summary for concrete manipulatives (Test Value = 1).

Item	Pre5	Pre6	Pre7	Pre8	PreRCMAv
Total N	59	59	59	59	59
Test Statistic	59.000	180.000	87.000	116.000	12.500
Standard Error	112.333	115.217	109.473	109.473	115.716
Standardized Test Statistic	−7.091	−6.119	−6.755	−6.490	−7.034
Asymptotic Sig.(2-sided test)	<.001	<.001	<.001	<.001	<.001

Table 8. One-sample wilcoxon signed rank test summary for virtual manipulatives (Test value = 1).

Item	Post1	Post2	Post3	Post4	PostVMAv
Total N	59	59	59	59	59
Test Statistic	1182.500	1510.500	1176.000	663.000	1173.000
Standard Error	101.041	106.637	103.827	90.156	102.296
Standardized Test Statistic	4.355	6.682	3.910	.283	5.235
Asymptotic Sig.(2-sided test)	<.001	<.001	<.001	.777	<.001

Table 9. One-sample wilcoxon signed rank test summary for concrete manipulatives (Test Value = 1).

Item	Post5	Post6	Post7	Post8	PostCMAv
Total N	59	59	59	59	59
Test Statistic	198.000	797.500	883.500	825.000	773.000
Standard Error	46.669	101.041	106.637	101.041	121.369
Standardized Test Statistic	−1.414	.544	.802	.816	−.206
Asymptotic Sig.(2-sided test)	.157	.586	.423	.414	.837

the difference by shading the figure and completing the addition sentence', the results ($p < 0.001$, $z = 4.355$) showed a statistically significant difference from the test value of 1, and the z-value is positive. Thus, students answered the first question correctly. The results of the second question, which asks students to 'Find the Difference between seven tenths and four tenths', indicated that there is a statistically significant difference from the test value ($p < .001$, $z = 6.682$). Since the z-value is positive, it is concluded that students could answer the second question correctly.

In the third question, which requires students to 'Find the difference of 4/5 - 1/5 - 2/5', the results ($p < 0.001$, $z = 3.910$) illustrated a statistically significant difference from the test value, which means that students answered the third question correctly. The results of the last question require respondents to write fractions such as 'Bilal ran the 5/7 of the distance in a race before stopping for a break. At the same time, Hessa ran 3/7 of the distance in the same race. Write the fraction that represents the difference between Bilal's and Hessa's distance in the race', it was not statistically significantly different from the test value ($p = 0.777 > 0.05$, $z = 0.283$). Therefore, the students could slightly answer the fourth question. The results of the average (PostVMAv) of the second four posttest questions related to learning fractions using virtual manipulatives were statistically significantly different from the test value ($p < 0.001$, $z = -5.235$) and the z-value is positive, which indicates that the students, in average, answered the four posttest questions related to virtual manipulatives strategy (See Table 8).

Posttest for concrete manipulatives

In Table 9, the one-sample Wilcoxon signed rank test was performed for four posttest questions related to concrete manipulatives. For the first question, which requires students to 'Find the Sum $1/5 + 5/10$', the results ($p = 0.157 > 0.05$, $z = -1.414$) showed that there was no statistically significant difference from the test value of 1. Thus, students partially answered the question. The results of the second question that requests students to 'Use the figure to find the sum of a half and one third and complete the addition sentence', indicated that there was no statistically significant difference from the test value ($p = 0.586 > 0.05$, $z = 0.544$), it was concluded that students could slightly answer the second question correctly.

In the third question that asks students to 'Find the Sum in simplest form $\frac{11}{12} + \frac{2}{3} + \frac{1}{4}$', the results ($p = 0.423 > 0.05$, $z = 0.802$) illustrated a non-statistically significant difference from the test value. Therefore, the students could partially answer the question. The results of the last question, which requires students to write fractions as follows 'Shaikha bought white chocolate and brown chocolate for her friends to celebrate teacher day at the school. $\frac{1}{4}$ of her friends chose the white chocolate while $\frac{3}{8}$

chose the brown chocolate. Write the fraction that represents the total number of students who took chocolate', was not statistically significantly different from the test value ($p = 0.414 > 0.05$, $z = 0.816$). Therefore, the students could partially answer the question. The results of the average (PostCMAv) of the posttest questions related to learning fractions (which were done by concrete manipulatives during the experiment) were not statistically significantly different from the test value ($p = 0.837 > 0.05$, $z = -0.206$), which indicates that the students, in average, partially answered the four posttest questions related to concrete manipulatives strategy (Table 9).

Group comparison (Mann-whitney U test) for the pre and posttest

The Mann-Whitney U test was utilized to compare the pre and posttest scores of students based on gender (Martin & Bridgmon, 2012).

Pretest for virtual manipulatives

An Independent-Samples Mann-Whitney U test was performed to examine if there was a statistically significant difference between the male and female students in their responses to the pretest related to learning fractions using virtual manipulatives (See Table 10). In the first question, students were asked to 'Show the difference by shading the figure and completing the subtraction sentence $\frac{5}{6}$ - $\frac{1}{6}$', the result showed that female students (Mean Rank = 32.09) performed better than male students (Mean Rank = 27.35). However, the difference was not statistically significant (Male: Mean Rank = 27.35, $N = 26$; Female: Mean Rank = 32.09, $N = 33$; $U = 498.000$, $Z = 1.616$, $p = 0.106 < 0.05$). In the second question, when students were asked to 'Find the Difference between seven tenths and four tenths', the result showed that male students (mean Rank = 30.65) performed better than the female students (mean Rank = 29.48), but the difference was not statistically significant (Male: Mean Rank = 30.65, $N = 26$; Female: Mean Rank = 29.48, $N = 33$; $U = 412.000$, $Z = -0.300$, $p = 0.764 < 0.05$) (See Table 10).

For the third question, which asks students to 'Find the difference of $\frac{4}{5}$ - $\frac{1}{5}$ - $\frac{2}{5}$', the result showed that male students (mean Rank = 32.81) performed better than female students (mean Rank = 27.79), but the difference was not statistically significant (Male: Mean Rank = 32.81, $N = 26$; Female: Mean Rank = 27.79, $N = 33$; $U = 356.000$, $Z = -1.879$, $p = 0.060 < 0.05$). Similarly, the fourth question asks, 'Bilal ran the $\frac{5}{7}$ of the distance in a race before stopping for a break. At the same time, Hessa ran $\frac{3}{7}$ of the distance in the same race. Write the fraction that represents the difference between Bilal's and Hessa's distance in the race', the result showed female students (mean Rank = 31.58) performed better than the male student (mean Rank = 28.00), but this difference was not statistically significant (Male: Mean Rank = 28.00, $N = 26$; Female: Mean Rank = 31.58, $N = 33$; $U = 481.000$, $Z = 1.822$, $p = 0.068 < 0.05$). The results of the average of the four questions related to learning fractions using virtual manipulatives showed that there was no statistically significant difference between male and female responses (Male: Mean Rank = 30.00, $N = 26$; Female: Mean Rank = 30.00, $N = 33$; $U = 429.000$, $Z = 0.000$, $p = 1.000 < 0.05$) (Table 10).

Pretest for concrete manipulatives

An Independent-Samples Mann-Whitney U test was performed to examine if there was a significant difference between the male and female students in their responses to the pretest related to learning fractions using concrete manipulatives. Results are shown in Table 11. With regard to the first question that requires students to 'Find the Sum $\frac{1}{5} + \frac{5}{10}$', the result showed that there was no statistically significant difference between the male and female responses (Male: Mean Rank = 30.75, $N = 26$; Female: Mean Rank = 29.41, $N = 33$; $U = 409.500$, $Z = -0.782$, $p = 0.434 < 0.05$). The second question that asks students

Table 10. Independent-samples Mann-Whitney U test summary for group comparison (Gender- Pre-RVM).

	Total N	Mann-Whitney U	Wilcoxon W	Test Statistic	Standard Error	Standardized Test Statistic	Male Mean Rank	Female Mean Rank	Asymptotic Sig.(2-sided test)
Pre1	59	498.000	1059.000	498.000	42.687	1.616	27.35	32.09	0.106
Pre2	59	412.000	973.000	412.000	56.580	−0.300	30.65	29.48	0.764
Pre3	59	356.000	917.000	356.000	38.845	−1.879	32.81	27.79	0.060
Pre4	59	481.000	1042.000	481.000	28.542	1.822	28.00	31.58	0.068
PreRVMAv	59	429.000	990.000	429.000	60.605	0.000	30.00	30.00	1.000

Table 11. Independent-samples Mann-Whitney U test summary for group comparison (Gender- Pre-RCM).

	Total N	Mann-Whitney U	Wilcoxon W	Test Statistic	Standard Error	Standardized Test Statistic	Male Mean Rank	Female Mean Rank	Asymptotic Sig. (2-sided test)
Pre5	59	409.500	970.500	409.500	24.934	−0.782	30.750	29.410	0.434
Pre6	59	389.000	950.000	389.000	34.294	−1.166	31.540	28.790	0.243
Pre7	59	494.000	1055.000	494.000	31.629	2.055	27.500	31.970	0.040
Pre8	59	415.500	976.500	415.500	34.337	−0.393	30.520	29.590	0.694
PreRCMAv	59	411.000	972.000	411.000	42.807	−0.420	30.690	29.450	0.674

Table 12. Independent-samples Mann-Whitney U test summary for group comparison (Gender- PostVM).

	Total N	Mann-Whitney U	Wilcoxon W	Test Statistic	Standard Error	Standardized Test Statistic	Male Mean Rank	Female Mean Rank	Asymptotic Sig. (2-sided test)
Post1	59	541.000	1102.00	541.00	50.986	2.197	25.69	33.39	0.028
Post2	59	469.000	1030.00	469.00	34.343	1.165	28.46	31.21	0.244
Post3	59	420.000	981.00	420.00	51.923	−0.173	30.35	29.730	0.862
Post4	59	563.500	1124.50	563.50	60.166	2.235	24.83	34.080	0.025
PostVMAv	59	581.500	1142.50	581.50	63.682	2.395	24.13	34.620	0.017

to 'Use the figure to find the sum of a half and one third and complete the addition sentence' showed that there was no statistically significant difference between the male and female responses the result (Male: Mean Rank = 31.54, N = 26; Female: Mean Rank = 28.79, N = 33; U = 389.000, Z = -1.166, $p = 0.243 < 0.05$).

For the third question, where students were required to 'Find the Sum in simplest form $\frac{11}{12} + \frac{2}{3} + \frac{1}{4}$', the result showed a statistically significant difference between male and female responses to the question (Male: Mean Rank = 27.50, N = 26; Female: Mean Rank = 31.97, N = 33; U = 494.000, Z = 2.055, $p = 0.040 < 0.05$) and that female student (mean Rank = 31.97) performed better than the male students (mean Rank = 27.50). The results of the fourth question related to 'Shaikha bought white chocolate and brown chocolate for her friends to celebrate teacher day at the school. $\frac{1}{4}$ of her friends chose the white chocolate while $\frac{3}{8}$ chose the brown chocolate. Write the fraction that represents the total number of students who took chocolate', indicated that there is no statistically significant difference between male and female responses to the question (Male: Mean Rank = 30.52, N = 26; Female: Mean Rank = 29.59, N = 33; U = 415.500, Z = -0.393, $p = 0.694 < 0.05$). The results of the average of the four questions related to learning fractions using virtual manipulatives showed that there is no statistically significant difference between male and female responses (Male: Mean Rank = 30.69, N = 26; Female: Mean Rank = 29.45, N = 33; U = 411.000, Z = -0.420, $p = 0.674 < 0.05$) (See Table 11).

Posttest for virtual manipulatives

In Table 12, the Independent-Samples Mann-Whitney U test was performed to examine if there were any statistically significant differences between the male and female students in their responses to the posttest related to learning fractions using virtual manipulatives. The first question asks students to 'Show the difference by shading the figure and completing the addition sentence $\frac{5}{6} - \frac{1}{6}$', (Male: Mean Rank = 25.690, N = 26; Female: Mean Rank = 33.390, N = 33; U = 541.000, Z = 2.197, $p = 0.028 < 0.05$). Thus, there is a statistically significant difference between the male and female responses. However, female students (mean Rank = 33.39) performed better than male students (mean Rank = 25.69). The second question, which requires students to 'Find the Difference between seven tenths and four tenths', showed that there is no statistically significant difference between the male and female responses (Male: Mean Rank = 28.460, N = 26; Female: Mean Rank = 31.210, N = 33; U = 469.000, Z = 1.165, $p = 0.244 < 0.05$). For the third question, when students were asked to 'Find the difference of $\frac{4}{5} - \frac{1}{5} - \frac{2}{5}$', the results showed that there was no statistically significant difference between male and female responses to the question (Male: Mean Rank = 30.350, N = 26; Female: Mean Rank = 29.730, N = 33; U = 420.000, Z = -0.173, $p = 0.862 < 0.05$). The results of the fourth question relate to 'Bilal ran the $\frac{5}{7}$ of the distance in a race before stopping for a break. At the same time, Hessa ran $\frac{3}{7}$ of the distance in the same race. Write the fraction that represents the difference between Bilal's and Hessa's distance in the race', indicating that there is a statistically significant difference between male and female responses (Male: Mean Rank = 24.830, N = 26; Female: Mean Rank = 34.080, N = 33; U = 563.500, Z = 2.235,

$p = 0.025 < 0.05$), and that female students (mean Rank = 34.08) performed better than male students (mean Rank = 24.83). The results of the average of the four questions related to learning fractions using virtual manipulatives showed that there is a statistically significant difference between male and female responses (Male: Mean Rank = 24.130, N = 26; Female: Mean Rank = 34.620, N = 33; U = 581.500, Z = 2.395, $p = 0.017 < 0.05$) and it is noticed that female students (mean Rank = 34.62) performed better than male students (mean Rank = 24.13). (See Table 12).

Posttest for concrete manipulatives

In Table 13, the Independent-Samples Mann-Whitney U test was performed to examine if there were any statistically significant differences between the male and female students in their responses to the posttest related to learning fractions using concrete manipulatives. In the first question that asks students to 'Find the Sum $\frac{1}{5} + \frac{5}{10}$', Table 13 showed that there is no statistically significant difference between the male and female responses (Male: Mean Rank = 25.710, N = 26; Female: Mean Rank = 33.380, N = 33; U = 540.500, Z = 1.839, $p = 0.066 < 0.05$), although female students (mean Rank = 33.38) performed better than male students (mean Rank = 25.71). In the second question, which requires students to 'Use the figure to find the sum of a half and one third and complete the addition sentence', the results showed that there is no statistically significant difference between the male and female responses (Male: Mean Rank = 30.810, N = 26; Female: Mean Rank = 29.360, N = 33; U = 408.000, Z = -0.357, $p = 0.721 < 0.05$).

For the third question, which is related to 'Find the Sum in simplest form $\frac{11}{12} + \frac{2}{3} + \frac{1}{4}$', the results showed that there is no statistically significant difference between male and female responses to the question (Male: Mean Rank = 30.770, N = 26; Female: Mean Rank = 29.390, N = 33; U = 409.000, Z = -0.346, $p = 0.729 > 0.05$). The results of the fourth question related to 'Shaikha bought white chocolate and brown chocolate for her friends to celebrate teacher day at the school. $\frac{1}{4}$ of her friends chose the white chocolate while $\frac{3}{8}$ chose the brown chocolate. Write the fraction that represents the total number of students who took chocolate', indicated there is no statistically significant difference between male and female responses to the question (Male: Mean Rank = 28.380, N = 26; Female: Mean Rank = 31.270, N = 33; U = 471.000, Z = 0.717, $p = 0.474 < 0.05$). The results of the average of the four questions related to learning fractions using virtual manipulatives showed that there is no statistically significant difference between male and female responses (Male: Mean Rank = 28.880, N = 26; Female: Mean Rank = 30.880, N = 33; U = 458.000, Z = 0.451, $p = 0.652 < 0.05$). (See Table 13).

Comparing pre/post tests for virtual manipulatives and concrete manipulatives

Comparisons between pre/posttest for virtual and concrete manipulatives were made using the effect size formula. According to Cohen (1988), the effect size is a quantitative measure that measures the strength of the experimental effect. A larger effect size indicates a stronger relationship between two variables. To compute the effect size between two groups, Goulet-Pelletier and Cousineau (2018) recommend subtracting the mean of one group from the other and dividing the result by the standard deviation of the population from which the groups were sampled. It is widely accepted that when the difference between the means of two groups is less than 0.2 standard deviations, the difference is trivial, even if it is statistically significant. On the other hand, if the d value equals or exceeds 0.8, the effect size is considered large, indicating a significant difference between the two parameters.

Table 14 shows that Cohen's d value for the correlation between pre and posttest questions related to learning fractions with virtual and concrete manipulatives is substantial, with values of 1.903 and

Table 13. Independent-samples Mann-Whitney U test summary for group comparison (Gender- PostCM).

	Total N	Mann-Whitney U	Wilcoxon W	Test Statistic	Standard Error	Standardized Test Statistic	Male Mean Rank	Female Mean Rank	Asymptotic Sig. (2-sided test)
Post5	59	540.50	1101.50	540.50	60.636	1.839	25.71	33.38	0.066
Post6	59	408.00	969.00	408.00	58.758	−0.357	30.81	29.36	0.721
Post7	59	409.00	970.00	409.00	57.808	−0.346	30.77	29.39	0.729
Post8	59	471.00	1032.00	471.00	58.614	0.717	28.38	31.27	0.474
PostCMAv	59	458.00	1019.00	458.00	64.324	0.451	28.88	30.88	0.652

Table 14. Cohen's d value and its effect size: Paired Samples Test.

Paired samples test										
		Paired Differences					t	df	Significance	Paired Samples Effect Sizes
					95% Confidence Interval of the Difference					
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			Two-Sided p	Standardizer Cohen's d
Pair 1	PostVMAv - PreRVMAv	1.12	0.59	0.08	0.97	1.27	14.62	58.00	0.00	1.903
Pair 2	PostCMAv - PreRCMAv	0.89	0.73	0.09	0.70	1.08	9.34	58.00	0.00	1.215
Pair 3	PostVMAv - PostCMAv	0.44492	0.63325	0.08244	0.27989	0.60994	5.397	58	0.000	0.703

1.215, respectively. These results indicate a statistically significant improvement in students' performance after learning fractions using both types of manipulatives. Moreover, the comparison between the two approaches indicates a statistically significant difference in students' progress, with a Cohen's d value of 0.703 indicating that using virtual manipulatives leads to significantly greater progress than using concrete manipulatives (Table 14).

Answering the research questions based on the results

Compared to the posttest answers after the intervention, for learning to subtract like fractions using virtual manipulatives, since there is no statistically significant difference from the median, 1, and the z-value is positive, students answered correctly three out of four questions. They answered partially correct the problem-solving. These results answered the first research question and rejecting the null hypothesis:

> **H_0 #1 (Q1)** There is no statistically significant difference in student achievement from the median value 1 (partially correct) in fraction addition and subtraction while teaching using virtual manipulatives at a significance level of $\alpha = 0.05$.

In learning to add, unlike denominators fractions using concrete manipulatives, there is no statistically significant difference for the four questions from the median, and the z-value is negative for the first question. This indicates that students failed to answer the first question and answered partially correctly to the other three questions. This answers the first research question and rejects the null hypothesis:

> **H_0 #2 (Q1)** There is no statistically significant difference in student achievement from the median value 1 (partially correct) in fraction addition and subtraction while teaching by using concrete manipulatives at a significance level of $\alpha = 0.05$.

The Paired Samples Test findings confirm the previous outcomes of the Independent-Sample Mann-Whitney U, indicating a significant enhancement in students' performance in fractions learning when using both types of manipulatives.

The Independent-Sample Mann-Whitney U test showed that in learning unlike fractions using virtual manipulatives, girls' answers were better than boys' in the first and the fourth question, the problem solving which answered the second research question by rejecting the hypothesis:

> **H_0 #3 (Q2)** There is no statistically significant difference at $\alpha \leq 0.05$ between the progress of male and female students in learning like fractions subtraction when using virtual manipulatives as a teaching method.

The results of the Paired Samples Test support the same results of the Independent-Sample Mann-Whitney U; that is, students' performance after learning fractions using both types of manipulatives was statistically significantly improved.

For the posttest questions on adding like denominators fractions using concrete manipulatives, there was no statistically significant difference between the male and female responses for all four questions, which answered the second research question by accepting the hypothesis:

H_0 #4 (Q2) There is no statistically significant difference at $\alpha \leq 0.05$ between the progress of male and female students in learning unlike fractions addition when using concrete manipulatives as a teaching method.

Cohen's d value for the correlation between pre and posttest questions related to learning fractions with virtual and concrete manipulatives is substantial. The results suggest a significant improvement in students' performance after learning fractions using both types of manipulatives. This answered the third research question by rejecting the null hypothesis:

H0 #5 (Q3) Based on the teaching strategy used, there is no evidence of a statistically significant difference in students' achievement in fractions' addition and subtraction at a significance level of $\alpha \leq 0.05$.

In addition, the comparison between the two approaches showed a significant difference in students' progress; that is, using virtual manipulatives leads to significantly more significant progress than using concrete manipulatives in learning fractions addition, and subtraction.

Discussion

The results of the One-Sample Test for the pretest questions related to learning subtracting like fractions using virtual manipulatives, students failed to answer 3 out of 4 questions and partially correct answered the fourth question. For the pretest questions related to learning adding unlike fractions using concrete manipulatives, students could not answer all four questions.

The study focused on the significance of employing diverse teaching approaches in mathematics, specifically on the topic of fractions. This topic has been identified as a challenging one for both teachers and students (Pantziara & Philippou, 2012). The results of this study suggest that incorporating virtual manipulatives as a teaching method can enhance students' innovative abilities (Kramarenko et al., 2019). Furthermore, and in line with the impact of virtual manipulatives on the learning of faction shown in this study, recent meta-analyses have indicated that the use of educational technology can positively impact learning outcomes (Reinhold, Hoch, Werner, Richter-Gebert, & Reiss, 2020). Therefore, it is crucial to involve students actively in the learning process and offer a variety of teaching methods, such as the ones employed in the present study, to promote engagement and enhance their overall learning experience.

Using concrete manipulatives (such as real models and pictures) to teach fractions provides students with a physical representation of the abstract concept of fractions. This approach makes the concept more tangible, allowing students to interact with the materials and develop a deeper understanding of the underlying principles of fractions. This view aligns with Moyer-Packenham et al. (2012) and Bouck, Mathews et al. (2020). By using manipulatives, students can see how fractions are formed and can experiment with different combinations and operations (Moyer-Packenham et al., 2012). This hands-on approach encourages active participation and engagement, which enhances learning and retention of the material. Additionally, manipulating physical objects allows students to visualize and understand fraction concepts, which can be challenging to grasp through verbal or written explanations alone (Jones & Tiller, 2017). Using concrete manipulatives to teach fractions helps students develop a more comprehensive understanding of the concept, encourages active participation, and enhances long-term material retention. Other studies (e.g. Reimer & Moyer, 2005; Vessonen et al., 2021) also confirmed the effectiveness of virtual manipulatives in learning mathematics, especially in development both conceptual and procedural understanding of fraction representations and operations.

While observing the results of some past studies, the findings about effectiveness of concrete and virtual manipulatives are mixed. For example, Mendiburo and Hasselbring (2014) reported that virtual manipulatives has almost same level of effect on students' learning as the concrete manipulatives. Likewise, Suh et al. (2005) found that virtual manipulatives were effective in discovery learning, conceptualizing mathematical relationships, representing fractions, and preventing general errors in fraction operations. These effects are also influenced by the context of using the online and virtual tools in teaching and learning fractions. For example, Park et al. (2024) reported students' improvement in their performance with varied outcomes that were influenced by student participation and their perception about virtual mode of instruction.

Drawing upon constructivist principles, students should not simply wait for instructions but instead engage in creative learning and construct knowledge through modeling and hands-on experiences (Golden, 2018). In the present study, using manipulatives allows students to become active participants and positively engaged in learning fractions via modeling and representation of fractions. Kocaoglu (2010) suggests that teachers can use models and shapes similar to those used with natural numbers to teach operations with fractions. This approach makes the concepts more tangible and easier to understand as the shapes and models provide a concrete representation. By relating fractions to students' everyday experiences, they can better comprehend and retain the information. Seeing fractions as part of their daily life makes the abstract concept more meaningful and leads to more lasting learning outcomes.

The study found that in two questions related to virtual manipulatives, girls performed better than boys. At the same time, no significant differences were observed between boys and girls on questions related to concrete manipulatives. One possible explanation for this difference is that girls are more engaged with technology and may have more experience using it outside the classroom. Additionally, girls tend to have more vital social and emotional skills, which can contribute to better collaboration and communication when working with technology. It is important to note that these findings should not be interpreted as a generalization that girls are always better at learning with technology than boys. Many factors can influence individual learning outcomes, and the results of this study may not apply to all contexts or populations. However, these findings suggest that teachers may want to consider incorporating technology into their teaching strategies to engage girls and boys in the learning process. For this, preservice teacher education should emphasize the conceptual development of future teachers in teaching and learning fractions (Alenazi, 2016; Thurtell et al., 2019). Effective teaching and learning strategies should be explored and implemented in elementary grades (Namkung & Fuchs, 2019) in order to develop representation and abstraction of fraction operations ((Nor'Arifahwati Abbas et al., 2022), beyond whole-part relation to the conceptual development of proportional reasoning (Çalisici, 2018).

Conclusion, limitations, and recommendations

This study provides valuable insights into the effectiveness of virtual and concrete manipulatives strategies for teaching fractions; it is important to consider its limitations, including the small sample size, limited scope, and a limited number of instructional periods used. Teaching fractions is considered one of the most challenging topics in mathematics. Students often forget what they have learned about fractions once they finished the final exams. The results of this study suggest that using virtual and concrete manipulatives as instructional methods for teaching fractions can provide various advantages for students. A significant benefit is the enhancement of their comprehension and performance in fractions, resulting in a positive transformation in their learning and attitudes towards not only fractions but also mathematics in general.

One of the limitations of this study is that the scope of this study was confined to four schools in Abu Dhabi Emirate. Therefore, the results may not apply to other schools within the same Emirate or other Emirates. Additionally, the study had a small sample size of only 59 fifth-grade students, which may limit the generalizability of the findings (Singer & Strasser, 2017). Another study limitation was that only one school followed the American curriculum, while the other three schools followed the United Arab Emirates' Ministry of Education curriculum. This means that there are many other schools in the United Arab Emirates that follow different curricula, such as British, Indian, Pakistani, Filipina, and so on. Including these schools in the study would provide a more comprehensive understanding of the impact of virtual and concrete manipulatives strategies on students' learning outcomes. Furthermore, the study had a limited number of instructional periods for teaching using virtual and concrete manipulatives, which may have influenced the effectiveness of the teaching method. The use of more instructional periods could potentially have yielded different results. Finally, if this study is repeated, it would be better to use different questions in the posttest than in the pretest. These questions should be parallel and at the same level to accurately measure the impact on students' performance.

After conducting this study, we recommend some crucial points for the teachers, the educational leaders, and the policymakers. First, it is essential for mathematics teachers to have regular training to

refresh, update, and gain new information about the subject knowledge, strategies of teaching, and the use of technology in teaching. As mentioned in many previous theories, such as constructivism, sociocultural theory, and Transformative learning theory, the role of the student in the learning process should be active; students have to lead their learning, and the teacher should be innovative in choosing the best method of teaching to engage all the students. Teaching fractions could be an exciting and enjoyable journey if the teachers did not consider it a burden. The first step in teaching fractions is planning. Planning starts by assessing the students' missed information that they cannot understand fractions without mastering this information such as operations with whole numbers, place value, and comparing and ordering numbers.

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Ethical approval

The research study that underpins this publication was provided by Social Sciences Ethics Sub-Committee in the United Arab Emirates University, Registration number ERS_2022_8451. The informed consent form was distributed to the students' guardians at school and obtained thereafter.

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